

Universal Bilinear Form of Quark and Lepton Mass Matrices

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Abstract

In the so-called “yukawaon” model, the (effective) Yukawa coupling constants Y_f^{eff} are given by vacuum expectation values (VEVs) of scalars Y_f (yukawaons) with 3×3 components. In the present model, all of the VEV matrices $\langle Y_f \rangle$ are given by a bilinear form of VEVs of flavons Φ_f , $\langle Y_f \rangle_i^j = k_f \langle \Phi_f \rangle_{ik} \langle \bar{\Phi}_f \rangle^{kj}$, where Φ_f is assigned to **6** of U(3) family symmetry. As input parameters with family-number dependent values, we use only charged lepton mass values. Under this formulation, we can give reasonable values of quark and lepton masses and their mixings. A CP violating phase $\delta_{CP}^\ell = 26^\circ$ in the lepton sector is predicted. The effective Majorana neutrino mass is also predicted.

PCAC numbers: 11.30.Hv, 12.15.Ff, 14.60.Pq, 12.60.-i,

1 Introduction

It is an interesting subject in the particle physics to investigate whether the observed hierarchical mass spectra and mixings of quarks and leptons result from a single origin or not. In this paper, we try to describe quark and lepton mass matrices by using only the observed values of charged lepton masses (m_e, m_μ, m_τ) as input parameters with family-number dependent values, and thereby, we investigate whether we can describe all other observed mass spectra (quark and neutrino mass spectra) and mixings (the Cabibbo-Kobayashi-Maskawa [1] (CKM) mixing and the Pontecorvo-Maki-Nakagawa-Sakata [2] (PMNS) mixing) without using any other family-number dependent parameters. Here, terminology “family-number independent parameters” means, for example, coefficients of a unit matrix **1**, a democratic matrix X_3 , and so on, where

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (1.1)$$

On the other hand, terminology “family-number dependent parameter” means, for example,

Table 1: Contrast of VEV relations in present yukawaon model to those in the previous yukawaon model [4]. For simplicity, notations “ \langle ” and “ \rangle ” are drop.

Previous	Present
$Y_e = k_e \Phi_0(\mathbf{1} + a_e X_3) \Phi_0$,	$Y_e = k_e \Phi_e \Phi_e$,
	$\Phi_e = k'_e \Phi_0(\mathbf{1} + a_e X_3) \Phi_0$,
$Y_\nu = k_\nu \Phi_0(\mathbf{1} + a_\nu X_2) \Phi_0$,	$Y_\nu = k_\nu \Phi_\nu \Phi_\nu + \xi_\nu \mathbf{1}$,
	$\Phi_\nu = k'_\nu \Phi_0(\mathbf{1} + a_\nu X_3) \Phi_0$,
$Y_u = k_u P_u \Phi_u \Phi_u P_u^\dagger$,	$Y_u = k_u \Phi_u \Phi_u + \xi_u \mathbf{1}$,
$\Phi_u = k'_u \Phi_0(\mathbf{1} + a_u X_3) \Phi_0$,	$P_u \Phi_u P_u = k'_u \Phi_0(\mathbf{1} + a_u X_3) \Phi_0$,
$Y_d = k_d \Phi_d \Phi_d$,	$Y_d = k_d \Phi_d \Phi_d$,
$\Phi_d = k'_d \Phi_0(\mathbf{1} + a_d X_3) \Phi_0 + \xi_d \mathbf{1}$,	$\Phi_d = k'_d \Phi_0(\mathbf{1} + a_d X_3) \Phi_0 + \xi'_d \mathbf{1}$,
$M_\nu = [Y_\nu Y_R^{-1} Y_\nu]^2$,	$M_\nu = Y_\nu Y_R^{-1} Y_\nu$,
$Y_R = Y_e \Phi_u + \Phi_u Y_e$,	$Y_R = Y_e \Phi_u + \Phi_u Y_e$,

coefficients of

$$\mathbf{1}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (1.2)$$

For such our purpose, in this paper, the investigation is done on the bases of the so-called yukawaon model [3, 4]. Here, the (effective) Yukawa coupling constants Y_f^{eff} are given by vacuum expectation values (VEVs) of scalars Y_f (yukawaons) with 3×3 components

$$(Y_f^{eff})_i^j = \frac{y_f}{\Lambda} \langle (Y_f)_i^j \rangle \quad (f = u, d, \nu, e), \quad (1.3)$$

where Λ is a scale of the effective theory. The conception of “yukawaons” are summarized as follows: (i) Yukawaons are a kind of flavons [5]. (ii) Those are singlets under the conventional gauge symmetries. (iii) Since yukawaons are fields, we can consider a non-Abelian family symmetry G by assigning suitable quantum numbers to Y_f . (In the present paper, we will assume $G=U(3)$.) (iv) The VEV forms are described by 3×3 matrices. (v) Each yukawaon is distinguished from others by R charges. (vi) VEV matrix relations are calculated from SUSY vacuum conditions. The relations are given by multiplicative forms among VEV matrices (e.g. $M_R = M_u^{1/2} M_e + M_e M_u^{1/2}$, and so on), differently from the conventional family symmetry models, in which mass matrix form is given by forms of additions (e.g. $M = c_1 M_1 + c_2 M_2 + \dots$). (vii) The VEV matrix $\langle Y_f \rangle$ also evolves after the family symmetry breaking in the same way that a conventional Yukawa coupling constant in the standard model (SM) evolves.

In order to see differences between the new model and the previous yukawaon model [4], we have listed the VEV relations of flavons in the present model in comparison to those in the previous yukawaon model in Table 1. Here, the VEV matrices Y_e , Y_ν , Y_u and Y_d correspond to charged lepton mass matrix M_e , neutrino Dirac mass matrix M_D , up-quark mass matrix M_u , and down-quark mass matrix M_d , respectively. For simplicity, we have dropped family indices although we consider family symmetries $U(3) \times U(3)'$. Also, notations “ \langle ” and “ \rangle ” were drop for simplicity.

VEV relations of flavons in the previous yukawaon model

Prior to describing of a new yukawaon model, let us give a brief review of the previous yukawaon model [4]. The essential VEV relations of flavons in the previous yukawaon model are listed in the left row in Table 1. As seen in Table 1, the previous yukawaon model has the following characteristics: (i) When we regard the form $\Phi_0(\mathbf{1} + a_f X_3)\Phi_0$ (Φ_0 is a diagonal VEV matrix) as one unit, Y_u and Y_d take bilinear forms, while Y_e and Y_ν are not so. (ii) Since $a_e \neq 0$, the VEV matrix Y_e is not diagonal. In an earlier version [6] of the yukawaon model, the VEV matrix Y_e was given by a bilinear form $Y_e = \Phi_e \Phi_e$ (Φ_e corresponds to Φ_0 in the previous model [4]), and thereby, a charged lepton mass relation [7]

$$K = \frac{m_e + m_\mu + m_\tau}{\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}} = \frac{2}{3}, \quad (1.4)$$

was speculated by considering a VEV matrix relation $\text{Tr}[\Phi_e \Phi_e] = \frac{2}{3} \text{Tr}[\Phi_e] \text{Tr}[\Phi_e]$. However, since the VEV matrix Y_e in the previous model do not take such a bilinear form, we cannot speculate the mass relation (1.4). The explanation of the formula (1.4) was one of the motivations of the yukawaon model. (iii) The matrix Y_ν contains the family-number dependent VEV matrix form X_2 which is defined in Eq.(1.2). The VEV matrix X_2 was brought in the model together with the unwelcome condition $a_e \neq 0$ in order to give the observed large neutrino mixing $\sin^2 2\theta_{13} \simeq 0.09$ [8]. However, our goal is a model without such a VEV matrix X_2 . (iv) Neutrino mass matrix M_ν was given by a double seesaw form (the so-called “inverse seesaw” form [9]) $M_\nu = (Y_\nu Y_R^{-1} Y_\nu)^2$, where Y_ν and Y_R are Dirac and Majorana neutrino mass matrices, respectively. The form was requested in order to give a reasonable ratio of neutrino squared mass difference, R_ν , which is defined in Eq.(3.14) later.

VEV relations of flavons in the present yukawaon model

The essential VEV relations of flavons in the present yukawaon model are listed in the right row in Table 1. The new model has the following characteristics: (i) VEV matrices of all yukawaons have the same family structure, while, in the previous yukawaon model, those were taken different forms for individual sectors. (ii) In the previous model, Y_e was not diagonal. However, in the new model, we succeed in building a model with $a_e = 0$, i.e. a charged lepton mass matrix with a diagonal form. In the new model, the VEV matrix Φ_e is diagonal, and given

by

$$\Phi_e = k'_e \text{diag}(m_e^{1/2}, m_\mu^{1/2}, m_\tau^{1/2}). \quad (1.5)$$

Therefore, we again has a possibility that the model leads to a charged lepton mass relation (1.4). (However, in this paper, we do not discuss the details.) (iii) In the previous model, in order to give a large value of lepton mixing parameter $\sin^2 2\theta_{13} \simeq 0.09$, we were obligated to bring an unwelcome VEV form Y_ν , i.e. a family-number dependent form $Y_\nu = \Phi_0(\mathbf{1} + a_\nu X_2)\Phi_0$. In contrast to the previous model, the present model has succeeded in removing such the family-number dependent VEV matrix form X_2 , and in unifying VEV matrix forms Φ_f into the form $\Phi_f = \Phi_0(\mathbf{1} + a_f X_3)\Phi_0$. (iv) Neutrino mass matrix is again simply taken as $M_\nu = Y_\nu Y_R^{-1} Y_\nu$ differently from $M_\nu = Y_\nu Y_R^{-1} Y_\nu \cdot Y_\nu Y_R^{-1} Y_\nu$ in the previous model.

We would like to emphasize that the purpose of the yukawaon model is to build a unified mass matrix model of quarks and leptons without introducing family-dependent parameters (as few as possible) except for the input values (m_e, m_μ, m_τ) . It is not our main purpose to build a model with economized parameters. Differently from conventional mass matrix model with a universal form (for a recent model, see, for example, Ref.[10]), we do not adhere to a universal form of mass matrices. In this paper, we propose a universal bilinear form of quark and lepton mass matrices. However, it is a by-product of our purpose, and our purpose itself is not to obtain a universal form of mass matrices.

In Sec.2, we will give details of the VEV matrix relations and superpotentials which give such VEV relations. In the yukawaon model, R charge assignments are essential for obtaining successful phenomenological results. Although we assign R charges from the phenomenological point of view, the assignments cannot be taken freely. We must take the assignments so that they may forbid appearance of unwelcome terms. The details are also discussed in Sec.2. In Sec.3, we give a parameter fitting under the new yukawaon model. Finally Sec.4 is devoted to a summary and concluding remarks.

2 Superpotential and VEV matrix relations

We assume that a would-be Yukawa interaction which is invariant under a family symmetry $U(3)$ is given as follows:

$$\begin{aligned} W_Y = & \frac{y_\nu}{\Lambda} (\nu^c)^i (\hat{Y}_\nu^T)_i{}^j \ell_j H_u + \frac{y_e}{\Lambda} (e^c)^i (\hat{Y}_e)_i{}^j \ell_j H_d + y_R (\nu^c)^i (Y_R)_{ij} (\nu^c)^j \\ & + \frac{y_u}{\Lambda} (u^c)^i (\hat{Y}_u)_i{}^j q_j H_u + \frac{y_d}{\Lambda} (d^c)^i (\hat{Y}_d)_i{}^j q_j H_d, \end{aligned} \quad (2.1)$$

where $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$ are $SU(2)_L$ doublets. The third term in Eq.(2.1) leads to the so-called neutrino seesaw mass matrix [11] $M_\nu = \hat{Y}_\nu Y_R^{-1} \hat{Y}_\nu^T$, where \hat{Y}_ν and Y_R correspond to neutrino Dirac and Majorana mass matrices, respectively. Here and hereafter, for convenience, we use notation \hat{A} , A and \bar{A} for fields with $\mathbf{8} + \mathbf{1}$, $\mathbf{6}$ and $\mathbf{6}^*$ of $U(3)$, respectively.

In order to distinguish each yukawaon from others, we assume that \hat{Y}_f have different R charges from each other together with considering R charge conservation (a global U(1) symmetry in $N = 1$ supersymmetry). (Of course, the R charge conservation is broken at an energy scale Λ , at which the U(3) family symmetry is broken.) For R parity assignments, we inherit those in the standard SUSY model, so R parities of yukawaons Y_f (and all flavons) are the same as those of Higgs particles (i.e. $P_R(\text{fermion}) = -1$ and $P_R(\text{scalar}) = +1$), while quarks and leptons are assigned to $P_R(\text{fermion}) = +1$ and $P_R(\text{scalar}) = -1$.

VEV relations among those yukawaons are obtained from SUSY vacuum conditions for superpotentials as we give later. Here, we need to introduce subsidiary flavons which have special VEV forms:

$$\langle E \rangle = \mathbf{1}, \quad \langle \bar{E} \rangle = \mathbf{1}, \quad (2.2)$$

$$\langle P_u \rangle = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1), \quad \langle \bar{P}_u \rangle = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1), \quad (2.3)$$

$$\langle \Phi_0 \rangle = \text{diag}(x_1, x_2, x_3), \quad \langle \bar{\Phi}_0 \rangle = \text{diag}(x_1, x_2, x_3), \quad (2.4)$$

$$\langle S_f \rangle = (\mathbf{1} + a_f X_3), \quad \langle \bar{S}_f \rangle = (\mathbf{1} + a_f X_3), \quad (2.5)$$

where we have dropped flavor-independent factors in those VEV matrices, because we deal with only mass ratios and mixings in this paper. The forms (2.4) and (2.5) are discussed later. (In (2.4) and (2.5), we have introduced another symmetry U(3)' in addition to the U(3) flavor symmetry.)

2.1 VEV forms of flavons E , \bar{E} , P_u , and \bar{P}_u

For flavons E and \bar{E} , we consider the following superpotential:

$$W_E = \lambda_{1E} \text{Tr}[E \bar{E} E \bar{E}] + \lambda_{2E} \text{Tr}[E \bar{E}] \text{Tr}[E \bar{E}], \quad (2.6)$$

where we have taken R charges such that

$$R(E) + R(\bar{E}) = 1. \quad (2.7)$$

The SUSY vacuum condition leads to

$$\langle E \rangle \langle \bar{E} \rangle = \mathbf{1}. \quad (2.8)$$

We choose a special solution of Eq.(2.8),

$$\langle E \rangle = \langle \bar{E} \rangle = \mathbf{1}. \quad (2.9)$$

For P_u and \bar{P}_u , we also consider the following superpotential form

$$W_P = \frac{\lambda_{1P}}{\Lambda} \text{Tr}[P_u \bar{P}_u P_u \bar{P}_u] + \frac{\lambda_{2P}}{\Lambda} \text{Tr}[P_u \bar{P}_u] \text{Tr}[P_u \bar{P}_u], \quad (2.10)$$

where we have taken R charges as

$$R(P_u) + R(\bar{P}_u) = 1. \quad (2.11)$$

The SUSY vacuum condition leads to

$$\langle P_u \rangle \langle \bar{P}_u \rangle = \mathbf{1}. \quad (2.12)$$

In general, it should be noted that for VEV matrices $\langle A \rangle$ and $\langle \bar{A} \rangle$ under the D -term condition, we can choose either one in two cases

$$\langle \bar{A} \rangle = \langle A \rangle^*, \quad (2.13)$$

$$\langle \bar{A} \rangle = \langle A \rangle. \quad (2.14)$$

We apply the case (2.13) to the VEV matrices $\langle P_u \rangle$ and $\langle \bar{P}_u \rangle$. Then, we obtain (2.3).

2.2 Superpotential forms of yukawaons \hat{Y}_f and sub-yukawaons Φ_f

Let us consider a superpotential for \hat{Y}_f ($f = \nu, e, u, d$),

$$W_{\hat{Y}} = \sum_{f=\nu, e, u, d} \left[\left(\mu_f (\hat{Y}_f)_i{}^j + \lambda_f (\Phi_f)_{ik} (\bar{\Phi}_f)^{kj} \right) (\hat{\Theta}_f)_j{}^i + \left(\mu'_f (\hat{Y}_f)_i{}^i + \lambda'_f (\Phi_f)_{ik} (\bar{\Phi}_f)^{ki} \right) (\hat{\Theta}_f)_j{}^j \right]. \quad (2.15)$$

Then, a SUSY vacuum condition $\partial W_{\hat{Y}} / \partial \hat{\Theta}_f = 0$ leads to VEV relation

$$\langle \hat{Y}_f \rangle = \langle \Phi_f \rangle \langle \bar{\Phi}_f \rangle + \xi_f \mathbf{1}, \quad (2.16)$$

where $\xi_f = \text{Tr} \left[[\langle \hat{Y}_f \rangle + \langle \Phi_f \rangle \langle \bar{\Phi}_f \rangle] \right]$. Here and hereafter, according to conventional yukawaon models, we have assume that all VEV matrices of the Θ flavons take $\langle \Theta \rangle = 0$. Therefore, SUSY vacuum conditions for other flavons do not bring any additional VEV relation.

Note that the appearance of $\xi_f \mathbf{1}$ terms in Eq.(2.16) is peculiar to the $\hat{\Theta}$ fields. If Θ fields have been $\mathbf{6}$ or $\mathbf{6}^*$ of $U(3)$, such a $\xi_f \mathbf{1}$ would not be able to appear. Meanwhile, as shown in Table 1, we have taken $\xi_e = \xi_d = 0$. The reason is purely based on a phenomenological requirement. (See the next section.)

For Φ_e and Φ_ν , we assume a superpotential

$$W_{\Phi_e, \Phi_\nu} = \sum_{f=e, \nu} \left(\mu_f (\Phi_f)_{ij} + \lambda_f (\Phi_0)_{i\alpha} (\bar{S}_f)^{\alpha\beta} (\Phi_0^T)_{\beta j} \right) (\bar{\Theta}_f)^{ji}, \quad (2.17)$$

which lead to

$$\langle \Phi_f \rangle = \langle \Phi_0 \rangle \langle \bar{S}_f \rangle \langle \Phi_0^T \rangle \quad (f = e, \nu), \quad (2.18)$$

where Φ_0 and S_f are new flavons which belong to $(\mathbf{3}, \mathbf{3})$ and $(\mathbf{1}, \mathbf{6}^*)$ of $U(3) \times U(3)'$, respectively. The VEV form of Φ_0 is given by Eq.(2.4). In general, we can choose the flavor basis such that $\langle \Phi_0 \rangle$ is diagonal. As we discuss later, since we take $a_e = 0$, we can denote Eq.(2.4) as

$$\langle \Phi_0 \rangle = \langle \bar{\Phi}_0 \rangle = \text{diag}(x_1, x_2, x_3) = \text{diag}(m_e^{1/4}, m_\mu^{1/4}, m_\tau^{1/4}), \quad (2.19)$$

from the D -term condition, where x_i are real and those are normalized as $x_1^2 + x_2^2 + x_3^2 = 1$. The VEV form of S_f is given by Eq.(2.5). We consider that the form (2.5) is due to a symmetry breaking $U(3)' \rightarrow S_3$ at $\mu = \Lambda'$. (Of course, we assume a superpotential similar to (2.17) for the flavons $\bar{\Phi}_f$).

On the other hand, for Φ_u , we assume a form

$$W_{\Phi_u} = \frac{1}{\Lambda} \left(\lambda_{1u} (\bar{P}_u)^{ik} (\Phi_u)_{kl} (\bar{P}_u)^{lj} + \lambda_{2u} (\bar{\Phi}_0)^{ik} (S_u)_{kl} (\bar{\Phi}_0^T)^{lj} \right) (\Theta_u)_{ji}, \quad (2.20)$$

which leads to

$$\langle \bar{P}_u \rangle \langle \Phi_u \rangle \langle \bar{P}_u \rangle = \langle \bar{\Phi}_0 \rangle \langle S_u \rangle \langle \bar{\Phi}_0^T \rangle. \quad (2.21)$$

In order to obtain $\xi'_d \mathbf{1}$ term for Φ_d as shown in Table 1, we assume the following superpotential

$$W_{\Phi_d} = \frac{\lambda_{1d}}{\Lambda} \text{Tr}[\bar{E} \Phi_d \bar{E} \Theta_d] + \frac{\lambda_{2d}}{\Lambda} \text{Tr}[\bar{\Phi}_0 S_d \bar{\Phi}_0^T \Theta_d] + \frac{\lambda_{3d}}{\Lambda} \text{Tr}[\bar{E} \Phi_d] \text{Tr}[\bar{E} \Theta_d], \quad (2.22)$$

which leads to

$$\langle \bar{E} \rangle \langle \Phi_d \rangle \langle \bar{E} \rangle = \langle \bar{\Phi}_0 \rangle \langle S_d \rangle \langle \bar{\Phi}_0^T \rangle + \xi'_d \langle \bar{E} \rangle, \quad (2.23)$$

where $\xi'_d = (\lambda_{3d}/\lambda_{1d}) \text{Tr}[\langle \bar{E} \rangle \langle \Phi_d \rangle]$. We can also consider a superpotential for $\bar{\Phi}_d$ accompanied with $\xi'_d \mathbf{1}$.

Note that in Eq.(2.22) we have added the λ_{3d} term to the λ_{1d} and λ_{2d} terms which correspond to the λ_{1u} and λ_{2u} terms in the superpotential W_{Φ_u} , Eq.(2.20). If we have considered a λ_{3u} term in W_{Φ_u} as well as the λ_{3d} term in W_{Φ_d} , we would obtain $\langle \bar{P}_u \rangle \langle \Phi_u \rangle \langle \bar{P}_u \rangle = \langle \bar{\Phi}_0 \rangle \langle S_u \rangle \langle \bar{\Phi}_0 \rangle + \xi'_u \langle \bar{P}_u \rangle$ with a complex coefficient $\xi'_u \propto \text{Tr}[\langle \bar{P}_u \rangle \langle \Phi_u \rangle]$ instead of Eq.(2.21). Then, not only the CKM parameters, but also the up-quark mass ratios and the PMNS parameters become dependent on the phase parameters (ϕ_1, ϕ_2) . We assume that the contribution from the λ_{3u} term is negligibly small from the practical reason for parameter fitting in the next section.

For Y_R , we assume a superpotential form

$$W_R = \left[\mu_R (Y_R)_{ij} + \lambda_R \left((\hat{Y}_e)_i^k (\Phi_u)_{kj} + (\Phi_u)_{ik} (\hat{Y}_e^T)_j^k \right) \right] (\bar{\Theta}_R)^{ji}, \quad (2.24)$$

which reads to

$$\langle Y_R \rangle = \langle \hat{Y}_e \rangle \langle \Phi_u \rangle + \langle \Phi_u \rangle \langle \hat{Y}_e^T \rangle. \quad (2.25)$$

The VEV relations described above have been derived dependently on the assignments of R charges for the flavons. The R charge assignments are discussed in the next subsection. In the

meanwhile, we list the assignments of $SU(2)_L \times SU(3)_c \times U(3) \times U(3)'$ for the fields which appear in the present model in Table 2. As seen in Table 2, the existence number of fields with $\mathbf{3}$ and $\mathbf{3}^*$ (and also $\mathbf{6}$ and $\mathbf{6}^*$) of $U(3)$ -family (and also $U(3)'$) are the same, so that the model are anomaly free.

2.3 R charge assignments

In this model, the existence number of flavons is larger than that of VEV relations. Therefore, in general, we can uniquely determine R charges of flavons. Since we make a request to assign R charges as simple as possible, we put the following rules:

(i) We assign the same R charge to flavons A and \bar{A} with the same VEVs, $\langle A \rangle = \langle \bar{A} \rangle$, e.g.

$$\begin{aligned} R(E) &= R(\bar{E}) = \frac{1}{2} \equiv r_E, \\ P(\Phi_0) &= R(\bar{\Phi}_0) \equiv r_0, \\ P(\Phi_f) &= R(\bar{\Phi}_f) \equiv r_f. \end{aligned} \tag{2.26}$$

Note that we consider $R(P_u) \neq R(\bar{P}_u)$ because of $\langle P_u \rangle \neq \langle \bar{P}_u \rangle$. Therefore, we obtain relations $R(S_u) = r_u + 2\bar{r}_P - 2r_0$ and $R(\bar{S}_u) = r_u + 2r_P - 2r_0$, separately. On the other hand, we take the option (2.14) for $\langle \Phi_\nu \rangle$, which contains a complex parameter a_ν as seen in the next section. Therefore, we take $\langle \Phi_\nu \rangle = \langle \bar{\Phi}_\nu \rangle$, so that $R(\Phi_\nu) = R(\bar{\Phi}_\nu) = r_\nu$. Then, $R(\hat{Y}_f)$ is simply given by

$$R(\hat{Y}_f) = 2R(\Phi_f) = 2r_f \quad (f = e, \nu, d, u), \tag{2.27}$$

from Eq.(2.16).

(ii) We can regard that R charges of \hat{Y}_f are determined only by those of the $SU(2)_L$ singlet fermions f^c . Therefore, we simply assign

$$R(\ell H_u) = R(\ell H_d) = R(q H_u) = R(q H_d) = 2. \tag{2.28}$$

(Since those have different quantum number of $U(1)_Y$, we can distinguish those from each other.) Then, we obtain a simple R charge relation

$$R(\hat{Y}_f) = -R(f^c). \tag{2.29}$$

For Y_R , we obtain

$$R(Y_R) = 2 - 2R(\nu^c) = 2 - 2 \left(2 - R(\ell H_u) - R(\hat{Y}_\nu) \right) = 2 + 2R(\hat{Y}_\nu), \tag{2.30}$$

from Eqs.(2.1) and (2.28). On the other hand, from Eq.(2.24), $R(Y_R)$ must be satisfied a relation

$$R(Y_R) = R(\Phi_u) + R(\hat{Y}_e). \tag{2.31}$$

If we consider $R(\hat{Y}_f) = 0$, then we can attach the field \hat{Y}_f on any term in superpotential. Therefore, we require $R(\hat{Y}_f) \neq 0$ for any $f = e, \nu, d, u$. Also, we have to require $R(\hat{Y}_f \hat{Y}_{f'}) \neq 0$

Table 2: Assignments of $SU(2)_L \times SU(3)_c \times U(3) \times U(3)'$. For R charges, see subsection 2.3. We assign the same R charges for flavons A and \bar{A} which have the same VEV $\langle A \rangle = \langle \bar{A} \rangle$, e.g. $R(A) = R(\bar{A})$. However, since $R(P_u) \neq R(\bar{P}_u)$ because of $\langle P_u \rangle \neq \langle \bar{P}_u \rangle$, we have $R(S_u) \neq R(\bar{S}_u)$ and $R(\Theta_u) \neq R(\bar{\Theta}_u)$, i.e. $r_{Su} = 2\bar{r}_P + r_u - 2r_0$, $\bar{r}_{Su} = 2r_P + r_u - 2r_0$, $r_{\Theta u} = 2 - 2r_0 - r_{Su}$, and $\bar{r}_{\Theta u} = 2 - 2r_0 - \bar{r}_{Su}$.

	ℓ	e^c	ν^c	q	u^c	d^c	H_u	H_d	\hat{Y}_e	\hat{Y}_ν	\hat{Y}_u	\hat{Y}_d
$SU(2)_L$	2	1	1	2	1	1	2	2	1	1	1	1
$SU(3)_c$	1	1	1	3	3*	3*	1	1	1	1	1	1
$U(3)$	3	3*	3*	3	3*	3*	1	1	8+1	8+1	8+1	8+1
$U(3)'$	1	1	1	1	1	1	1	1	1	1	1	1
R	2	$-2r_e$	$-2r_\nu$	2	$-2r_u$	$-2r_d$	0	0	$2r_e$	$2r_\nu$	$2r_u$	$2r_d$

Y_R	$\bar{\Phi}_e$	Φ_e	$\bar{\Phi}_\nu$	Φ_ν	$\bar{\Phi}_u$	Φ_u	$\bar{\Phi}_d$	Φ_d	\bar{P}_u	P_u	Φ_0	$\bar{\Phi}_0$
1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1
6	6*	6	6*	6	6*	6	6*	6	6*	6	3	3*
1	1	1	1	1	1	1	1	1	1	1	3*	3
r_R	r_e	r_e	r_ν	r_ν	r_u	r_u	r_d	r_d	$1 - r_P$	r_P	r_0	r_0

S_e	\bar{S}_e	S_ν	\bar{S}_ν	S_u	\bar{S}_u	S_d	\bar{S}_d
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
6	6*	6	6*	6	6*	6	6*
$r_e - 2r_0$	$r_\nu - 2r_0$	r_{Su}	\bar{r}_{Su}	$r_d + 1 - 2r_0$			

E	\bar{E}	$\hat{\Theta}_e$	$\hat{\Theta}_\nu$	$\hat{\Theta}_u$	$\hat{\Theta}_d$	$\bar{\Theta}_R$
1	1	1	1	1	1	1
1	1	1	1	1	1	1
6	6*	8+1	8+1	8+1	8+1	6*
1	1	1	1	1	1	1
$\frac{1}{2}$	$\frac{1}{2}$	$2 - 2r_e$	$2 - 2r_\nu$	$2 - 2r_u$	$2 - 2r_d$	$2 - 2r_R$

Θ_{Φ_e}	$\bar{\Theta}_{\Phi_e}$	Θ_{Φ_ν}	$\bar{\Theta}_{\Phi_\nu}$	Θ_{Φ_u}	$\bar{\Theta}_{\Phi_u}$	Θ_{Φ_d}	$\bar{\Theta}_{\Phi_d}$
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
6	6*	6	6*	6	6*	6	6*
1	1	1	1	1	1	1	1
$2 - r_e$		$2 - r_\nu$		$r_{\Theta u}$	$\bar{r}_{\Theta u}$	$1 - r_d$	

for any combination of f and f' . As a result, we have to consider that whole R values of \hat{Y}_f are positive. Therefore, we speculate that the values of R will be describe by simple integers, so that, by way of trial, let us put

$$\left(R(\hat{Y}_\nu), R(\hat{Y}_u), R(\hat{Y}_e), R(\hat{Y}_d)\right) = (1, 2, 3, 4). \quad (2.32)$$

Then, the assignments (2.32) give

$$\begin{aligned} R(Y_R) &= 2 + 2r_\nu = 2 + 2 = 4, \\ R(\Phi_u) + R(\hat{Y}_e) &= r_u + 2r_e = 1 + 3 = 4, \end{aligned} \quad (2.33)$$

so that the requirement (2.31) is satisfied. Note that, thus, the simple assignment of R , Eq.(2.32), guarantees the existence of the flavon interaction term (2.24), which plays a very important role in giving the peculiar form of neutrino Majorana mass matrix.

3 Parameter fitting

3.1 How many parameters?

We summarize our mass matrices M_f as follows:

$$M_e = [\Phi_0(\mathbf{1} + a_e X_3)\Phi_0]^2 + \xi_e \mathbf{1} \quad (a_e = 0, \xi_e = 0), \quad (3.1)$$

$$M_D = [\Phi_0(\mathbf{1} + a_\nu e^{i\alpha_\nu} X_3)\Phi_0]^2 + \xi_\nu \mathbf{1}, \quad (3.2)$$

$$M_u = P_u ([\Phi_0(\mathbf{1} + a_u X_3)\Phi_0]^2 + \xi_u \mathbf{1}) P_u^*, \quad (3.3)$$

$$M_d = [\Phi_0(\mathbf{1} + a_d X_3)\Phi_0 + \xi'_d \mathbf{1}]^2, \quad (3.4)$$

$$M_\nu = M_D Y_R^{-1} M_D, \quad Y_R = Y_e \Phi_u + \Phi_u Y_e. \quad (3.5)$$

Here, for convenience, we have dropped the notations “ \langle ” and “ \rangle ”. Since we are interested only in the mass ratios and mixings, we use dimensionless expressions $\Phi_0 = \text{diag}(x_1, x_2, x_3)$ (with $x_1^2 + x_2^2 + x_3^2 = 1$), $P_u = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, and $E = \mathbf{1} = \text{diag}(1, 1, 1)$. Therefore, the parameters a_e, a_ν, \dots are re-defined by Eqs.(3.1)-(3.5).

Meanwhile, we require “economy of the number of parameters”. Namely, we neglect parameters which play no essential roles in numerical fitting to the mixings and mass ratios as far as possible. In the present model, we assume that the parameters a_e, a_u and a_d are real, while a_ν is complex. So that we have denoted the parameter a_ν as $a_\nu e^{i\alpha_\nu}$ in Eq.(3.2). We also assume that the parameters ξ_f ($f = e, u$, and ν) and ξ'_d are real. We consider that the charged lepton sector is the most fundamental flavor scheme, and the charged lepton mass matrix should take the most simple form. Therefore, we assume $a_e = 0$ and $\xi_e = 0$ in Eq.(3.1). Then, the parameter values x_1/x_2 and x_2/x_3 are fixed by the charged lepton masses as

$$\frac{x_1}{x_2} = \left(\frac{m_e}{m_\mu}\right)^{1/4}, \quad \frac{x_2}{x_3} = \left(\frac{m_\mu}{m_\tau}\right)^{1/4}. \quad (3.6)$$

So we obtain

$$(x_1, x_2, x_3) = (0.115144, 0.438873, 0.891141), \quad (3.7)$$

where we have normalized x_i as $x_1^2 + x_2^2 + x_3^2 = 1$.

Therefore, in the present model, except for the parameters (x_1, x_2, x_3) , we have 9 adjustable parameters, $(a_\nu, \alpha_\nu, \xi_\nu)$, (a_u, ξ_u) , (a_d, ξ'_d) , and (ϕ_1, ϕ_2) for the 16 observable quantities (6 mass ratios in the up-quark-, down-quark-, and neutrino-sectors, four CKM mixing parameters, and 4+2 PMNS mixing parameters). Especially, quark mass matrices M_u and M_d are fixed by two parameters (a_u, ξ_u) and (a_d, ξ'_d) , respectively. (Note that those parameters are family-number independent parameters.) Therefore, in order to fix those parameters, we use two input values, up-quark mass ratios $(m_u/m_c, m_c/m_t)$ and down-quark mass ratios $(m_d/m_s, m_s/m_b)$, respectively, as we discuss in the next subsection 3.2. After the parameters (a_u, ξ_u) and (a_d, ξ'_d) have been fixed by the observed quark mass ratios, we have five parameters $(a_\nu, \alpha_\nu, \xi_\nu)$ and (ϕ_1, ϕ_2) as remaining free parameters. Processes for fitting those five parameters are listed in Table 3. In subsection 3.3, we discuss PMNS mixing ($\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, and $\sin^2 2\theta_{13}$) and neutrino mass ratio ($R_\nu \equiv \Delta m_{21}^2 / \Delta m_{32}^2$) by adjusting three parameters $(a_\nu, \alpha_\nu, \xi_\nu)$. Also, in subsection 3.4, we discuss four CKM mixing parameters, $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and $|V_{td}|$, by adjusting two parameters (ϕ_1, ϕ_2) .

Note that the purpose of the present paper is not to compete with other models for reducing parameter number in the model, but to investigate whether it is possible or not to fit all of the mixing parameters and mass ratios without using any family number dependent parameters when we use only the observed charged lepton masses as family dependent parameters. If we pay attention only to fitting of mixing parameters, a model with fewer number of parameters based on quark-lepton complementarity [12] is rather excellent compared with the preset model. (For such a recent work, see, for example, Ref.[13] and references there in.)

3.2 Quark mass ratios

From the observed values [14]

$$r_{12}^u \equiv \sqrt{\frac{m_u}{m_c}} = 0.045^{+0.013}_{-0.010}, \quad r_{23}^u \equiv \sqrt{\frac{m_c}{m_t}} = 0.060 \pm 0.005, \quad (3.8)$$

at $\mu = m_Z$ [14], we fix values of (a_u, ξ_u) . We find four solutions of (a_u, ξ_u) which can give the values (3.8). Only one solution

$$(a_u, \xi_u) = (-1.467, -0.001467), \quad (3.9)$$

can give a reasonable prediction of the PMNS mixing as we discuss later.

From the observed down-quark mass ratios [14]

$$r_{12}^d \equiv \frac{m_d}{m_s} = 0.053^{+0.005}_{-0.003}, \quad r_{23}^d \equiv \frac{m_s}{m_b} = 0.019 \pm 0.006, \quad (3.10)$$

Table 3: Process for fitting parameters. $N_{parameter}$ and N_{input} denote a number of free parameters in the model and a number of observed values which are used as inputs in order to fix these free parameters, respectively. $\sum N_{...}$ means $\sum N_{parameter}$ or $\sum N_{input}$

Step	Inputs	N_{input}	Parameters	$N_{parameter}$	Predictions
1st	$m_e/m_\mu, m_\mu/m_\tau$	2	$x_1/x_2, x_2/x_3$	2	—
	$m_u/m_c, m_c/m_t$	2	a_u, ξ_u	2	—
	$m_d/m_s, m_s/m_b$	2	a_d, ξ'_d	2	—
2nd	$\sin^2 2\theta_{12}, \sin^2 2\theta_{23}, R_\nu$	3	$\xi_\nu, a_\nu, \alpha_\nu$	3	$\sin^2 2\theta_{13}, \delta_{CP}^\ell$ 2 Majorana phases, $\frac{m_{\nu 1}}{m_{\nu 2}}, \frac{m_{\nu 2}}{m_{\nu 3}}$
3rd	$ V_{cb} , V_{ub} $	2	(ϕ_1, ϕ_2)	2	$ V_{us} , V_{td} , \delta_{CP}^q$
option	Δm_{32}^2		$m_{\nu 3}$		$(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}), \langle m \rangle$
$\sum N_{...}$		11		11	

we determine the parameters (a_d, ξ'_d) as follows:

$$(a_d, \xi'_d) = (-1.477, +0.0237). \quad (3.11)$$

3.3 PMNS mixing

The observed values [15] are

$$\begin{aligned} \sin^2 2\theta_{12} &= 0.857 \pm 0.024, \\ \sin^2 2\theta_{23} &> 0.95, \end{aligned} \quad (3.12)$$

$$R_\nu \equiv \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{m_{\nu 2}^2 - m_{\nu 1}^2}{m_{\nu 3}^2 - m_{\nu 2}^2} = \frac{(7.50 \pm 0.20) \times 10^{-5} \text{ eV}^2}{(2.32_{-0.08}^{+0.12}) \times 10^{-3} \text{ eV}^2} = (3.23_{-0.19}^{+0.14}) \times 10^{-2}. \quad (3.13)$$

First, we fix the parameter ξ_ν as $\xi_\nu = -0.020$ so as to reproduce reasonable values (3.12) and (3.13). Next, we determine the parameter values of $(a_\nu, \alpha_\nu, \xi_\nu)$ as follows:

$$(a_\nu, \alpha_\nu, \xi_\nu) = (3.53, 8.7^\circ, -0.020). \quad (3.14)$$

Here the values of $(a_\nu, \alpha_\nu, \xi_\nu)$ in Eq. (3.14) are obtained so as to reproduce the observed values of the PMNS mixing angles and R_ν . We show the a_ν and α_ν dependences of the PMNS mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and R_ν in Fig. 1(a) and Fig. 1(b), respectively. It is found that R_ν is very sensitive to a_ν .

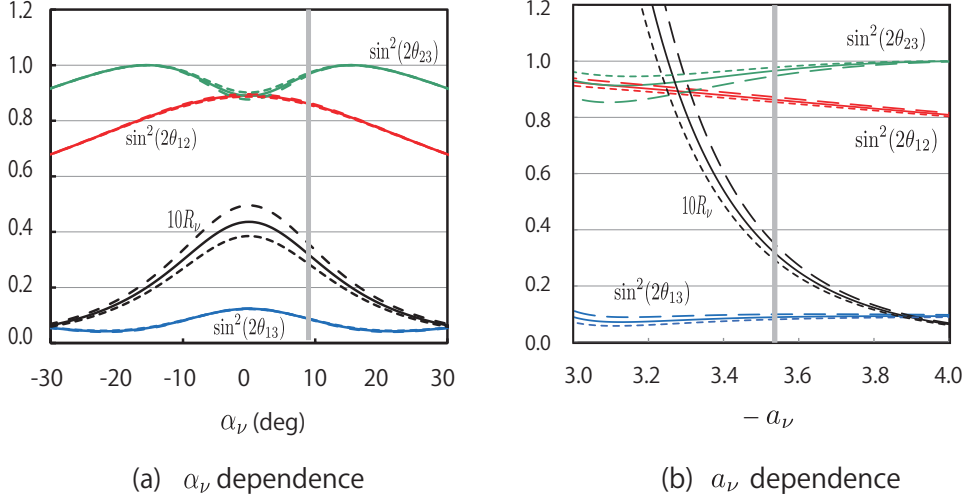


Figure 1: (a): α_ν dependence of the lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared difference ratio R_ν . We draw curves of those as functions of α_ν for the case of $\xi_\nu = -0.20$ with taking $a_\nu = -3.5$ (dotted), -3.53 (solid), and -3.56 (dashed). (b): a_ν dependence of the lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared difference ratio R_ν . We draw curves of those as functions of a_ν for the case of $\xi_\nu = -0.20$ with taking $\alpha_\nu = 7.0^\circ$ (dotted), 8.7° (solid), and 10° (dashed).

3.4 CKM mixing

Next, we discuss quark sector. Since the parameters (a_u, ξ_u) and (a_d, ξ'_d) have been fixed by the observed quark mass ratios, the CKM mixing matrix elements $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$ are functions of the remaining two parameters ϕ_1 and ϕ_2 . In Fig. 2, we draw allowed regions in the (ϕ_1, ϕ_2) parameter plane which are obtained from the observed constraints of the CKM mixing matrix elements shown in Eq. (3.15), with taking $\xi_u = -0.001467$, $a_u = -1.467$, $a_d = -1.477$, and $\xi'_d = 0.0237$. As shown in Fig. 2, all the experimental constraints on CKM parameters are satisfied by fine tuning the parameters ϕ_1 and ϕ_2 around

$$(\phi_1, \phi_2) = (21.8^\circ, -4.9^\circ). \quad (3.15)$$

Here we have used the observed values [15]

$$\begin{aligned} |V_{us}| &= 0.22534 \pm 0.00065, \\ |V_{cb}| &= 0.0412^{+0.0011}_{-0.0005}, \\ |V_{ub}| &= 0.00351^{+0.00015}_{-0.00014}, \\ |V_{td}| &= 0.00867^{+0.00029}_{-0.00031}. \end{aligned} \quad (3.16)$$

3.5 Neutrino masses and leptonic Dirac CP violating phase

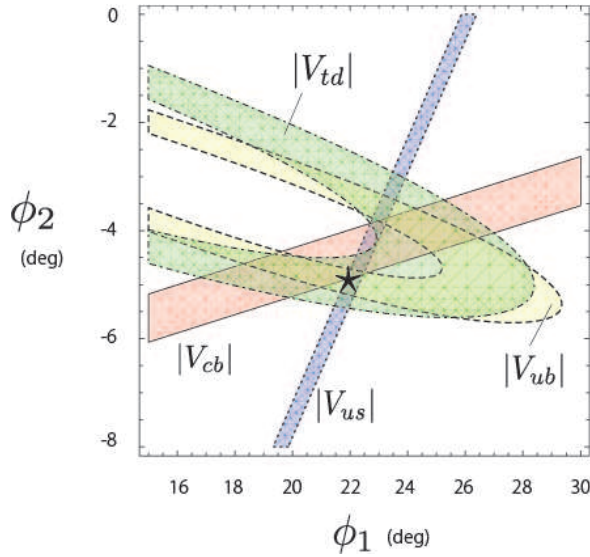


Figure 2: Allowed region in the (ϕ_1, ϕ_2) parameter plane obtained by the observed values of the CKM mixing matrix elements $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$. We draw allowed regions obtained from the observed constraints of the CKM mixing matrix elements shown in Eq. (3.15), with taking $\xi_u = -0.001467$, $a_u = -1.467$, $a_d = -1.477$, and $\xi'_d = 0.0237$. Here we take 2σ errors for all the observed values of the CKM mixing matrix elements. We find that the parameter set around $(\phi_1, \phi_2) = (21.8^\circ, -4.9^\circ)$ indicated by a star (\star) is consistent with all the observed values.

We can predict neutrino masses, for the parameters given by (3.9) and (3.14), as follows

$$m_{\nu 1} \simeq 0.00040 \text{ eV}, \quad m_{\nu 2} \simeq 0.00890 \text{ eV}, \quad m_{\nu 3} \simeq 0.0501 \text{ eV}, \quad (3.17)$$

by using the input value [16] $\Delta m_{32}^2 \simeq 0.00241 \text{ eV}^2$.

We also predict the effective Majorana neutrino mass [17] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = |m_{\nu 1}(U_{e1})^2 + m_{\nu 2}(U_{e2})^2 + m_{\nu 3}(U_{e3})^2| \simeq 5.1 \times 10^{-3} \text{ eV}. \quad (3.18)$$

Our model also predicts $\delta_{CP}^\ell = 25.7^\circ$ for the Dirac CP violating phase in the lepton sector, which indicates relatively large CP violating effect in the lepton sector. (Note that the previous model predicts $\delta_{CP}^\ell = 179^\circ$ which indicates small CP violating effect in the lepton sector.)

4 Concluding remarks

We have tried to describe quark and lepton mass matrices by using only the observed values of charged lepton masses (m_e, m_μ, m_τ) as input parameters with family-number dependent values. Thereby, we have investigated whether we can describe all other observed mass spectra (quark and neutrino mass spectra) and mixings (CKM and PMNS mixings) without using any

Table 4: Predicted values vs. observed values.

	$ V_{us} $	$ V_{cb} $	$ V_{ub} $	$ V_{td} $	δ_{CP}^q	r_{12}^u	r_{23}^u	r_{12}^d	r_{23}^d
Pred	0.2225	0.0430	0.00405	0.00800	55.8°	0.0416	0.0627	0.0492	0.0192
Obs	0.22534	0.0412	0.00351	0.00867	68°	0.045	0.060	0.053	0.019
	± 0.00065	$+0.0011$ -0.0005	$+0.00015$ -0.00014	$+0.00029$ -0.00031	$+10^\circ$ -11°	$+0.013$ -0.010	± 0.005	$+0.005$ -0.003	$+0.006$ -0.006
	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	$R_\nu [10^{-2}]$	δ_{CP}^ℓ	$m_{\nu 1} [\text{eV}]$	$m_{\nu 2} [\text{eV}]$	$m_{\nu 3} [\text{eV}]$	$\langle m \rangle [\text{eV}]$
Pred	0.863	0.965	0.089	3.25	25.7°	0.00040	0.00890	0.0501	0.00514
Obs	0.857	> 0.95	0.095	3.23	-	-	-	-	$< O(10^{-1})$
	± 0.024		± 0.010	$+0.14$ -0.19					

other family-number dependent parameters. In conclusion, as seen in Sec.3, we have obtained reasonable results. Our predicted values are listed in Table 4.

However, we have been still obliged to bring a family-number dependent VEV matrix P_u given in Eq.(2.3). When we consider that our aim has been completed except for only P_u , and that it appears only in the quark sector, there is a possibility that the origin of the matrix form P_u is not due to a VEV form of a flavon P_u , but it may be due to another origin, for example, a dynamical origin such as QCD effects, and so on. This is an open question at present.

In the present revised version of yukawaon model, the following points are worthy of note:

(i) We have been able to describe the VEV matrices of the yukawaons with the unified forms $\hat{Y}_f = \Phi_f \bar{\Phi}_f$.

(ii) Especially, we have adopted a bilinear form for charged lepton mass matrix, $\hat{Y}_e = \Phi_e \bar{\Phi}_e$. It is for the first time to succeed in giving a large value $\sin^2 2\theta_{13} \sim 0.09$ without taking a non-diagonal form of \hat{Y}_e . By this model-change, the charged lepton mass formula (1.4) has again become possible to understand from $\text{Tr}[\Phi_e \bar{\Phi}_e] = \frac{2}{3} \text{Tr}[\Phi_e] \text{Tr}[\bar{\Phi}_e]$ although we did not discuss the relation (1.4) in the present paper.

(iii) The VEV relation of Y_R to Φ_u and \hat{Y}_e , Eq.(2.25), is ad hoc assumption in the past models [3, 4]. (The R -charges have been assigned so that the ad hoc relation $R(Y_R) = R(\Phi_u) + R(\hat{Y}_e)$ may be satisfied.) In the present model, we have demonstrated that a simple R charge assignment (2.32) guarantees the relation (2.25). At present, the meaning of the assignment (2.32) is unclear, investigation of which is left to our future task.

(iv) In the present model, we have predicted the CP violating phase in the lepton sector as $\delta_{CP}^\ell \simeq 26^\circ$, which is sufficiently large to observe CP violation effects in future experiments. (In the previous model [4], a predicted value of δ_{CP}^ℓ was $\delta_{CP}^\ell \simeq 179^\circ$, which was invisibly small.) The origin of the CP violation is in the phase factor α_ν in the Dirac neutrino mass matrix (3.2). Note that we have taken $\alpha_f = 0$ ($f = e, u, d$) for economy of the parameters. However, we have been obliged to accept $\alpha_\nu \neq 0$ in order to fit the observed value of $\sin^2 2\theta_{13}$.

We still have some open questions as follows:

(a) Compared with the previous model [4], number of free parameters is not so reduced in the present yukawaon model. As emphasized in Sec.1, the purpose of the present paper is not to build a model with economized parameters. In the present yukawaon model, the VEV relations among flavons have been given by universal forms compared with those in the past yukawaon models [3]. Some of the parameters in the past yukawaon models have been eliminated, but, instead, terms which shift VEV matrices of yukawaons by unit matrices $\xi_f \mathbf{1}$ (or $\xi'_f \mathbf{1}$) have been newly added in the present model. This means that the present model cannot give predictions as far as the mass ratios are concerned, and it is nothing but that two parameters (a_f and ξ_f) or (a_d and ξ'_d) are fixed by the two observed mass ratios. Therefore, in the present model, only mixings can be predicted as far as quark sector is concerned.

(b) In spite of our aim to describe whole of quark and lepton masses and mixings by using only the observed charged lepton masses as input parameters with hierarchical values, we again need family-number dependent parameters (ϕ_1, ϕ_2) in the description of the CKM mixing. Also the origin of CP violation in the quark sector is in the phase matrix P_u , i.e. the phase parameters (ϕ_1, ϕ_2). [Note that in the lepton sector the origin of $\delta_{CP}^\ell \neq 0$ is $\alpha_\nu \neq 0$ which is inevitably required in order to get reasonable fitting of the PMNS mixing angles and the neutrino mass ratio R_ν .] Namely, we have different origins of CP violations between lepton and quark sectors. This is still unsatisfactory to us. The phase matrix P_u has family-number dependent parameters (ϕ_1, ϕ_2), so that such parameters should be eliminated in the final goal of the yukawaon model. We consider that, in a yukawaon model at the final goal, the CP violation in the quark sector, too, should be brought by family-number independent parameters α_u, α_d , and so on.

By success of the present major improvement of the yukawaon model, it seems that we are considerably close to the ideal stage that all hierarchical structures of quarks and leptons can be understood only from the family-number dependent parameter values (m_e, m_μ, m_τ). However, at present, we have many flavons and free parameters. Our next task is to economize numbers of those flavons and free parameters.

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